

## Stepper Motor Technical Note

### Determining Torque Sensitivity And Holding Torque Of Two Phase Permanent Magnet Stepper Motors

Frequently the torque sensitivity of a two-phase stepper motor is not specified in the vendor's published literature. This important, intrinsic stepper motor parameter is easily calculable.

One method to determine the torque sensitivity is utilizing the specification or measurement of the back emf in volts/(k steps/sec) and the full step angle. Often, a stepper motor vendor specifies the back emf and step angle. In any case, both are easily ascertained by testing.

#### Derivation

#### SYMBOLS AND UNITS

Symbol	Definition	Units
$\Delta$	Step Angle-Full Step	$^{\circ}/\text{step}^*$
$I$	Current in a Phase	Amp
$I_{1\emptyset}$	Maximum Current- 1 Phase On	Amp
$I_{2\emptyset}$	Maximum Current- 2 Phase On	Amp
$K$	Motor Constant	Volts-sec <u>or</u> Nm/Amp
$K_E$	Back emf constant	Volts/(k steps/sec)
$K_{ER}$	Back emf constant	Volts-sec**
$K_T$	Torque Sensitivity	Nm Amp***
$R_H$	Phase Resistance at Maximum Winding Temperature	Ohm
$R_{TH}$	Thermal Resistance-Winding To Ambient	$^{\circ}\text{C}/\text{Watt}$
$T_{AMB}$	Ambient Temperature	$^{\circ}\text{C}$
$T_{MAX}$	Maximum Winding Temperature	$^{\circ}\text{C}$
$T_H$	Holding Torque generated by an Individual Phase with Current $I_{2\emptyset}$	Nm
$T_{H1\emptyset}$	Maximum Holding Torque-1 Phase On	Nm
$T_{H2\emptyset}$	Maximum Holding Torque-2 Phase On	Nm
$\Delta T$	Temperature Drop, Winding to Ambient	$^{\circ}\text{C}$

\* degrees/step      \*\* "Volts-sec" is equivalent to "Volt/(rad/sec)"

\*\*\* "Nm/Amp" is Newton-meters/Amp

### To Compute Torque Sensitivity

To determine the back emf  $K_{ER}$  if we know  $K_E$  is merely an issue of changing the units from "Volts/(k steps/sec)" to "Volts/(rad/sec)"!

$$1. K_{ER} = [K_E \text{ Volts}/(\text{k steps}/\text{sec})] \times [1/\text{Å}^\circ/\text{step}] \times (360^\circ/\text{rev}) \times (1 \text{ rev}/2 \pi \text{ rad})$$

Simplifying equations 1:

$$2. K_{ER} = K_E \times 0.18 / (\pi \times \text{Å})$$

When the back emf is in "Volt-sec" the value of the torque constant is in "Nm/A", having the same numerical value.

A proof of this is in the Appendix I, at the end of this document. Therefore:

$$3. K_{ER} = K_T$$

#### Example:

The ARSAPE two-phase, 10 mm diameter stepper motor AM1020-A-0.25-7.

Step angle =  $18^\circ$

Back emf  $K_E = 1.5 \text{ Volts}/(\text{k steps}/\text{sec})$

Utilizing equation 2:

$$K_{ER} = K_E \times 0.18/(\pi \times \text{Å}) = 1.5 \times .18 / (\pi \times 18^\circ)$$

$$K_{ER} = 4.775 \times 10^{-3} \text{ Volt-sec}$$

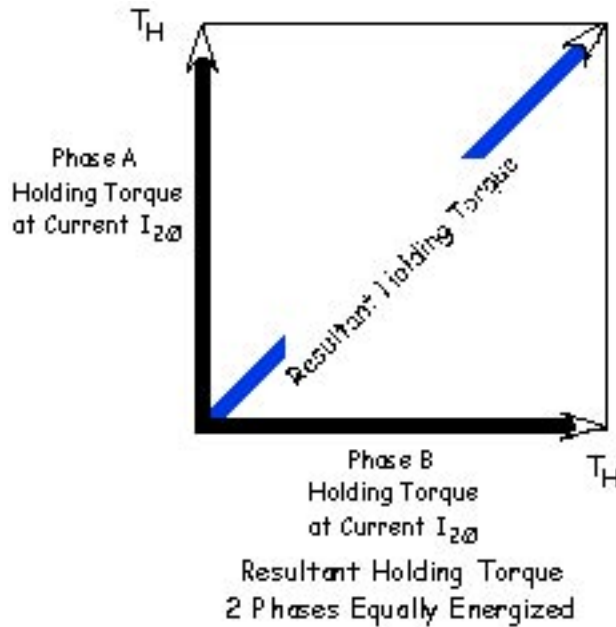
Utilizing equation 3:

$$K_T = K_{ER} = 4.775 \times 10^{-3} \text{ Nm/A}$$

### To Compute Holding Torque- 2 Phase On

If both phases are equally energized Figure 1 illustrates that the resulting torque of the two phases is the  $\sqrt{2}$  times the holding torque of one phase.

We will use the maximum rated phase current for two phases energized  $I_{2\phi}$ , calculate the holding torque of each phase, and determine a Resultant Holding Torque, which in this case will be  $T_{H2\phi}$ , the Maximum Holding Torque-2 Phase On.



**Figure 1**

The holding torque generated by each energized phase, when both are equally energized, is:

$$4. \quad T_H = K_T \times I_{2\theta}$$

For two phases, as illustrated in Figure 1, the Resultant Holding Torque is:

$$5. \quad T_{2\theta} = \sqrt{2} \times T_H$$

$$6. \quad T_{H2\theta} = \sqrt{2} \times K_T \times I_{2\theta}$$

The AM1020-A-0.25-7 is rated for 0.25 Amps, two phases on.

Utilizing equation 7:

$$T_{H2\theta} = \sqrt{2} \times K_T \times I_{2\theta}$$

$$T_{H2\theta} = \sqrt{2} \times 4.775 \times 10^{-3} \text{ Nm/A} \times 0.25 \text{ Amps}$$

Therefore the holding torque, for two phases on is:

$$T_{H2\theta} = 1.69 \times 10^{-3} \text{ Nm}$$

### To Compute Holding Torque- 1 Phase On

We will utilize the same example motor from above for this example. The 10 mm diameter two phase stepper motor AM1020-A-0.25-7 is rated for 0.25 Amps, two phases on.

For one phase on only we can increase this current by the  $\sqrt{2}$  and have the same power dissipation.

A proof of this is in the Appendix II, at the end of this document.

$$I_{1\theta} = \sqrt{2} \times I_{2\theta}$$

$$I_{1\theta} = \sqrt{2} \times 0.25 \text{ Amps}$$

$$I_{1\theta} = 0.354 \text{ Amps}$$

So the rated current is 0.354 Amps with one phase on.

$$T_{H1\emptyset} = K_T \times I_{1\emptyset}$$

$$T_{H1\emptyset} = 4.775 \times 10^{-3} \text{ Nm/A} \times 0.354 \text{ Amps}$$

$$T_{H1\emptyset} = 1.69 \times 10^{-3} \text{ Nm}$$

Notice that the Maximum Holding Torque is the same, one phase on or two phases on, at their respective maximum current ratings, assuming that magnetic saturation has not been reached.

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## Appendix I

Proof that the back emf constant in "volt-sec" is numerically equal to the torque sensitivity in "Nm/A".

This is simply a change of units, and illustrates the beauty of the MKS System.

$$K = 1 \text{ Volt-sec}$$

We have the following definitions:

$$\text{Volt} = 1 \text{ Joule/Coulomb}$$

$$\text{Coulomb} = 1 \text{ Amp-second}$$

$$\text{Joule} = 1 \text{ Newton-meter} = \text{Nm}$$

Substituting and rearranging where necessary:

$$K = 1 \text{ Volt-sec} = 1 \text{ sec (Joule/Coulomb)}$$

$$K = 1 \text{ sec (Nm/Coulomb)} = 1 \text{ sec-Nm/ (Amp-sec)} = 1 \text{ Nm/ Amp}$$

$$K = 1 \text{ Nm/A}$$

## Appendix II

Determination of Maximum Current with One Phase On vs. Two Phases On.

Assume that the maximum current for Two Phases On is specified as  $I_{2\phi}$ .

The heat flow

$$A. \quad \Delta T = T_{MAX} - T_{AMB}$$

Or

$$B. \quad \Delta T = Q R_{TH}$$

This is "Ohm's Law of heat flow. When we have 2 phases the heat generated is the sum of the heat generated in each phase:

$$B. \quad Q_{2\phi} = I_{2\phi}^2 R_H + I_{2\phi}^2 R_H = 2 I_{2\phi}^2 R_H$$

When only one phase is energized the heat generated is solely from one phase or:

$$C. \quad Q_{1\phi} = I_{1\phi}^2 R_H$$

To maintain the same  $\Delta T$  the dissipation with one phase energized must equal the dissipation with two phases energized or:

$$D. \quad \Delta T = (I_{1\phi}^2 R_H) R_{TH} = (2 I_{2\phi}^2 R_H) R_{TH}$$

Solving for  $I_{1\phi}$

$$I_{1\phi}^2 = (2 I_{2\phi}^2 R_H) R_{TH} / (R_H R_{TH}) = 2 I_{2\phi}^2$$

$$I_{1\phi} = \sqrt{2} I_{2\phi}$$